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MATHEMATICAL MODEL FOR BLOOD FLOW (USING POISEUILLE LAW)

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ABSTRACT :

This paper presents mathematical model for flow of blood by application of Poiseuille law. Infact blood is carried from the heart to several parts of the body through a system of elastic tuber e.g. the arteries, capillaries and veins and eventually returned to the heart without leaving the system. Here to form a simple mathematical model for blood flow in arteries, we assume that blood is homogeneous, the flow is steady and laminar and the tubes (arteries) are rigid, long and straight. Thus under these assumptions, poiseuille's law is applicable. We have also provided discussion and limitation of the model along with properties of blood and bifurcation in an artery.

Key Words : Elastic tubes, artery, steady, laminar, veins.

1. Introduction

Various researchers have shown their keen interest in mathematical models of blood flow through arteries in recent years [1-7, 8, 9]. The flow of blood in large arteries is considered as Newtonian flow [10]. Newtonian flow is defined as that flow where shear stress is directly proportional to shear rate. This means that when shear stress is plotted against shear rate at a given temperature the plot shows a straight line with a constant slop, this slop is called the viscosity of fluid, also Newtonian flow can be defined as the flow in which the coefficient of viscosity is constant [11]. Aneurysm is an enlarged size of arteries caused by a weakening of the arteries wall. This weakening can lead to rupture of blood artery at aneurysm location, which can cause internal bleeding and stooped the blood supply. For instance, rupture an aneurysm in the artery which is supplying the blood to brain, can bring strokes, likewise rupture in abdominal artery leads to death from internal bleeding [12]. The relation between the aneurysm and hemodynamic of blood flow is considered the interest subject of researchers Konods et al. [13] have suggested that, irregular shear stress of wall is one of the factors which contribute to the weakening of othe wall tissue. Shojima et al. [14] have reported that low wall shear stress is related to the growth of aneurysm, and rupture. Loic et al. [15] have observed that the aneurysm growth occur in regions of low wall shear stress. Some other worker's in this field are Herman [16], Sah and Siddiqi [17], Verma et al. [18] etc.

In this paper we have discussed mathematical models for blood flow using Poiseuille's law. We have also provided here interpretation and limitations of the model along with properties of blood and biburfcation in an artery.

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2. Some Basic Concepts

(A) Fluid Parameters :

Approaching the problem from a macroscopic (continuum approach) point of view by assuming that any small volume element of the fluid is so large that it contains a very large number of molecules and also assuming that the fluid is a continuous medium, the dependent ariables describing the fluid motion are we take parameters as follow:

(i) Pressure $P \equiv P\ (x, y, z, t)$

(ii) Density $P \equiv P\ (x, y, z, t)$

(iii) Velocity $\overline{q} = \overline{q}(x, y, z, t)$

If the density □ is taken to be constant throughout the flow, then the flow is said to be incompressible

(B) Viscosity:

Newton defined the viscosity of a fluid as a lake of slipperiness between the lawyers of the fluid and of course, in doing so he implied that was such a thing as a "layer" or lamina of fluid, and the viscosity as a result of rubbing one lamina upon the other.

Let us consider two layers which are in contact with one another at a distance y and y + dy from x – axis.

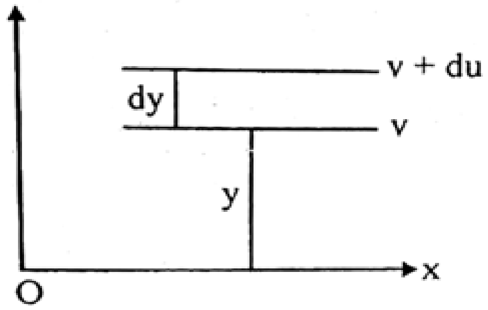

Fig. 1

Taking some force f parallel to the x-axis acts and produces relative motion between the two layer i.e., upper layer moves with velocity du relative to the lower layer. Then by Newton's hypothesis,

$$F \propto A \frac{du}{dy}$$

$$F = \mu A \frac{du}{dy} \qquad \text{........(i)}$$

Where □ is the proportionality constant which gives the measure of the viscosity of the fluid □ is also called the coefficient of viscosity and A is the area of content between the layers.

As the equation (i) is a linear relation, thus those fluid which follows the newton's hypothesis given by (1) are called Newtonian fluid. Newtonian fluids are often divided up into two categories viz. viscous compressible and viscous incompressible.

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(C) Poiseuille's Flow

In 1840, Poiseuille first considered the flow through a circular cylindrical tube and, got a relation between the flow rate and pressure gradient. This relation is known as Poiseuille's law.

Here we consider the fluid flow through a circular tube of length *l* and diameter 2a which is small compared to length. We consider that the rate of flow is steady. We further take that the fluid velocity everywhere inside the tube is laminar i.e., the velocity is fully in the direction of the axis of the tube, it is zero at the inner surface and maximum on this axis.

Let us take the axis of the cylinder (tube) as z0axis, so we use the cylindrical coordinates (r, □, z). Let a_r, $q_{\square}$ and q_z be the velocity component of the fluid. Since the flow is along z-axis only so that

$$q_r = 0, \; q_{\square} = 0$$

From the symmetry of the problem, q_z does not depend on the polar angle □ in the plane perpendicular to the axis of z, so that

(2.1) $q_z = q_z\,(r, z)$

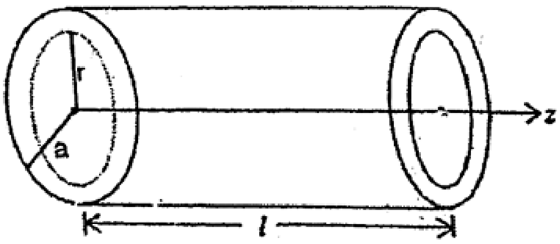

Fig. 2

Now from equation of continuity, we have

(2.2) $$\frac{\partial q_z}{\partial z} = 0$$

Equation (2.2) gives that q_z is not a function of z

(2.3) $$q_z = q_z(r)$$

We have from Navier stoke equations

$$\frac{\partial p}{\partial r} = 0$$

Which implies that p = p (z)

Now from z component of the Navier-Stokes equation, gives

$$\rho\left(\frac{\partial q_z}{\partial z} + q_z \frac{\partial q_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu \nabla^2 q_z$$

Which by using equation (2.3) simplifies to

(2.4) $$\mu \nabla^2 q_z = \frac{\partial p}{\partial z}$$

The ∇^2 operator in cylindrical coordinates is given by

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$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

Therefore, with the help of (2.3), equation (2.4) now becomes :

(2.5) $$\mu \frac{1}{r}\frac{d}{dr}\left(r\frac{dq_z}{dr}\right) = \frac{\partial p}{\partial z}$$

Since p = p (z), then equation (2.5) can be written as

(2.6) $$\mu \frac{1}{r}\frac{d}{dr}\left(r\frac{dq_z}{dr}\right) = \frac{\partial p}{\partial z}$$

As q_z is a function of r only and p is a function of z only, then from a separation of variables argument we conclude that each term of equation (2.6) is constant.

$$\therefore \frac{dp}{dz} = \text{constant}$$

Let us denote this constant pressure gradient by P

$$\therefore \frac{dp}{dz} = -P$$

Now, equation (2.6) becomes

(2.7) $$\frac{1}{r}\frac{d}{dr}\left(r\frac{dq_z}{dr}\right) = -\frac{P}{\mu}$$

On integrating (2.7) w.r.t.r, we have

(2.8) $$r\frac{dq_z}{dr} = -\frac{P}{2\mu}r^2 + A$$

Dividing both sides by r and integrating again yields

(2.9) $$q_z = -\frac{Pr^2}{4\mu} + A\log r + B$$

Since q_z at r = 0 (on the axis) is finite, therefore, we have A = 0. Also form the no-slip condition i.e., $q_z = 0$ at r = a, we have

(2.10) $$B = \frac{Pa^2}{4\mu}$$

$$\therefore q_z(r) = \frac{P}{4\mu}(a^2 - r^2)$$

This expression gives the velocity at a far r from the axis and also gives zero on the tube’s surface and a maximum along the axis.

From equation (2.11) it is clear that the velocity profile is parabolic in r and q_z and in the three dimensional space, it may be regarded as a paraboloid of revolution.

The volume flux (flud discharge), or the whole volume of the fluid crossing any section per unit of time, is denoted by Q and is given by

(2.11) $$Q = \int_0^a 2\pi r q_z dr = \frac{\pi a^4 P}{8\mu}$$

This result gives that the rate of flow proportional to the pressure gradient. Thus, this formula is known as Poiseuille's law. If one end of the cylindrical tube is at pressure p_1 and the other and is at pressure p_2 where $p_1 > p_2$. Then the formula given in equation (2.11) becomes.

(2.12) $$Q = \frac{\pi}{8}\frac{(p_1 - p_2)}{\mu I} a^4$$

This formula has the following properties :

(i) $Q \propto a^4$

(ii) $Q \propto p_1 - p_2$

(iii) $Q \propto \frac{1}{\ell}$

The above results were discovered experimentally by Hagen in 1839. The "fourth power law" is often invoked to show that if you narrow the tube even slightly the pressure difference required to keep the flow constant must increase greatly.

For example if the arteries becomes slightly constricted, the blood pressure required to supply blood adequately will rise considerably, leading to a state of hypertension.

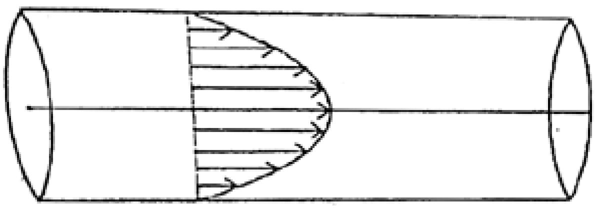

Fig. 3

3. Model of Blood Flow

We know that blood is carried from the heart to various parts of the body through a system of elastic tubes, e.g., the arteries, capillaries and veins and eventually returned to the heart without leaving the system. This process is known as circulation of blood or flow of blood.

In order to form a mathematical model, we first identify the essential characteristics of the blood flow. It is a non-homogeneous fluid and blood pots are elastic and having branches repeated. Also Flow of Blood is unsteady and in general laminar except for flow near the heart.

(I) Formulation of Model

Since the above essential characteristics of the blood flow will make the model very complicated therefore, we now formulate a simple mathematical model for blood flow in arteries by making certain assumption.

(i) Blood is a homogenous fluid.

(ii) The flow is steady and laminar.

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(iii) The tubes (arteries) are rigid, long and straight.

Thus, under these assumption, Poiseuille law is applicable. Therefore, the velocity of the blood in such a configuration is

(3.1) $$q_z(r) = \frac{(p_1 - p_2)}{4\mu l}(a^2 - r^2), 0 \le r \le a$$

And the rate of flow is

(3.2) $$Q = \frac{\pi a^4 (p_1 - p_2)}{8\mu l}$$

If τ_{rz} is the shear stress, then by Newton formula, we have

$$\tau_{rz} = \mu\left[\frac{\partial q_z}{\partial r}\right] = \mu\left[\frac{(p_1 - p_2)}{4\mu l}(-2r)\right]$$

(3.3) $$\tau_{rz} = \frac{-r(p_1 - p_2)}{2l}$$

The shear stress on the wall, is

$$\tau_{rz} = \frac{-a(p_1 - p_2)}{2l}$$ [putting r = a in]

(II) Remarks :

Since the rate of the blood flow is

$$Q = \frac{\pi a^4 (p_1 - p_2)}{4\mu l}$$

Obviously the rate of the flow depends on 1, $p_1 - p_2$ a and □.

The for a given p and for given constant □, if the radius of the arteries is decreased to half the mark, then the flow of the blood in the arteries reduces to sixteenth of the original value which effect the regulation of normal metabolic activities of the body.

If $(q_z)_{av} = \frac{Q}{\pi a^2}$ is the average velocity over the cross section, then

$$p_1 - p_2 = \frac{8\mu l}{a^2}(q_z)_{av}$$

Example : To calculate viscosity of blood for each of the following sets of flow data.

Radius of Blood Vessel	Length	Mean velocity of Blood	Pressure Difference
3×10^{-4} cm	0.075 cm	0.25 cm/s	2×10^4 dyne/cm^2
3×10^{-2} cm	3.0 cm	5.0 cm/s	4×10^3 dyne/cm^2

Solution : We know that

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$$P_1 - P_2 = \frac{8\mu l}{a^2}(q_z)_{av} \quad \text{.........(1)}$$

(i) $a = 3 \times 10^{-4}$ cm, $1 = 0.075$ cm, $(q_z)_{av} = 0.25$ cm/s
and $p_1 - p_2 = 2 \times 10^4$ dyne/cm^2

putting these values in (1), we get

$$\mu = \frac{2\times10^4\times(3\times10^{-4})^2}{8\times0.075\times0.25} = \frac{18\times10^{-4}}{8\times1875\times10^{-5}}$$

$$= \frac{180}{15000} = 0.012$$

(ii) $a = 3 \times 10^{-2}$ cm, $1 = 3.0$ cm $(q_z)_{av} = 5.0$ cm/s
and $p_1 - p_2 = 4 \times 10^3$ dyne/cm2

putting these values in (1), we get

$$\mu = \frac{4\times10^3\times(3\times10^{-2})^2}{8\times3\times5} = \frac{36\times10^{-1}}{120} = \frac{36}{1200} = 0.03 \text{ poise}$$

III. Limitation of the Model : During the formulation of the model we have made certain assumptions, these assumptions are satisfied only partly.

Therefore we have the following limitations over the assumption

(i) We have assumed that blood has been taken homogeneous which obeys the Newton's law of friction. If we consider the small blood vessels, the cell structure should be included resulting non-linear terms of high degree in model.

(ii) We have assumed, flow to be steady because the flow in all large arteries and the intothoracic veins is shown to be pulsative which is infact a time dependent flow and not steady in case, thus the above model is not applicable in such cases.

(iii) Flow is assumed to be laminar which is not taken always because at rate of flow above critical value, the flow becomes turbulent. Also the flow near the heart and aorta is pulsative that is flow is unsteady there.

4. Properties of Blood

For Newtonian viscous fluids, we have

□ = □e

where □ is the shear stress and e is the shear strain rate. Simple model for non-Newtonian behavior is the power law model given by

$$\tau - \mu e^n$$

$$= \mu e^{n-1} e,$$

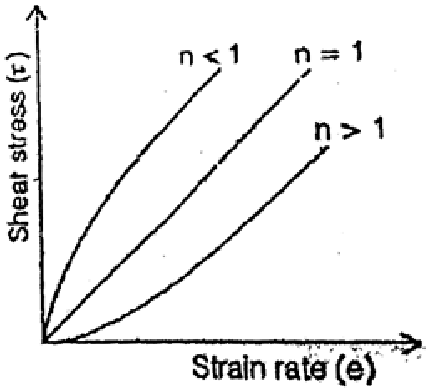

Fig. 4

Where □e^{n-1} is effective viscosity coefficient. If $n < 1$, then the fluid is Pseudo-plastic-power fluid.

If $n = 1$, then the fluid is Newtonian fluid.

If $n > 1$, then the fluid is dilatants fluid.

Most of the biological fluids are in fact non-Newtonian. Blood contains of a suspension of cells in an aqueous solution called plasma which is composed of 90% water and 7% protein. When blood is subjected to centrifugal force in a centrifuge (a powerful instrument for spinning biochemical solutions at very large angular velocities), it separates out into plasma and formed elements namely blood cells and platelets.

The blood cells consists of red cell (or erythrocytes) which spreads oxygen from the lungs to all cells, and white cell (or leukocytes) which play an important active in the resistance of the body to infections. Platelets from 5% of the total and they performs a function related to blood clotting.

Blood plasma is found to behave like a normal Newtonian fluid. Thus, the Non-Newtonian of blood is direct consequence of the fact that blood is a suspension of cells in plasma.

Bifurcation in an Artery

A bifurcation is where the artery branches into two smaller arteries during embroyogenesis. With the help of the principle of minimization of energy dissipation in conjuction with Posiseuille's law we can determine the angle of branching and also we can locate the bifurcation. In order to study the bifurcation, we use the following theory.

For steady flow in a tube, if the loss of K.E. (kinetic energy) of the blood in going from the inlet to the outlet is left, then the dissipated power P is equal to the work done by the pressure forces at the inlet and outlet ends, which implies that

P = Q□p

where p is the dissipated power, Q the rate of flow of blood through the artery and □P the pressure drop down to its length.

Let us assume a symmetrical bifurcation when a mother artery is divided into two equal daughter arteries as shown below.

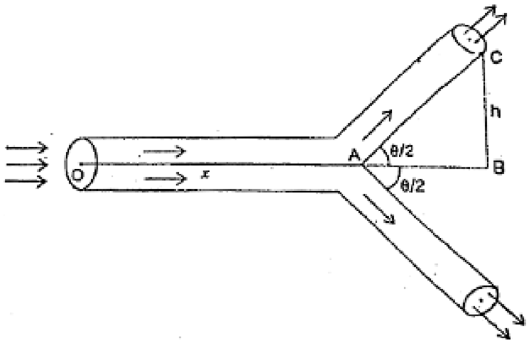

Fig. 5

Let $2a_1$ be the diameter of the mother artery and a_2 the diameter of each daughter artery and let p_1 and p_2 be the pressure in usual meanings respectively and p_x be the pressure at bifurcation position A and Q_1 and Q_2 the blood flux in the main artery and bifurcated arteries respectively.

Then the sum of total dissipated power is given by

$$P = Q_1(p_1 - p_x) + Q_2(p_x - p_2) + Q_2(p_x - p_2)$$

(4.1) or $P = Q(p_1 - p_x) + 2Q_2(p_x - p_2)$

By the conservation of flux, we have

$Q_1 = 2Q_2$

Equation (4.1) becomes

(4.2) $P = Q_1 (p_1 - p_2)$

by, Poiseuille's law, we have

$$Q_1 = \frac{\pi(p_1 - p_x)}{8\mu} OAa_1^4$$

(4.3) or $Q_1 = \frac{\pi(p_1 - p_x)}{8\mu x} a_1^4 (\therefore OA = x)$

Again by Poiseuille's law, we have

$$Q_2 = \frac{\pi}{8\mu} \frac{(p_1 - p_x)a_1^4}{AC}$$

Let OB = L so that AB = L – x, therefore in $\Delta ABC \angle CBA = 90^o$ and BC = h, so $AB = \sqrt{\left[h^2 + (L-x)^2\right]}$

(4.4) $\therefore Q_2 = \frac{\pi}{8\mu} \frac{(p_x - p_2)a_2^4}{\sqrt{h^2 + (L-x)^2}}$

Combining (4.3), (4.4) and $Q_1 = 2Q_2$ for P_x we get

(4.5) $p_x = \frac{p_1 a_1^4 \sqrt{h^2 + (L-x)^2} + 2p_2 a_2^4 x}{a_1^4 \sqrt{h^2 + (L-x)^2} + 2a_2^4 x}$

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Using (4.2) and (4.3) and (4.5) we obtain

$$\text{(4.6)} \quad P = \frac{\pi}{4\mu} \frac{a_1^4 a_2^4 (p_1 - p_2)^2}{a_1^4 \sqrt{h^2 + (L-x)^2} + 2a_2^4 x}$$

5. Discussion

Equation (4.6) shows that P is a function of x, therefore, for the minimum value of P, we must have

$$\frac{dP}{dx} = 0$$

$$\text{(5.1)} \quad \Rightarrow x = L - \frac{2a_2^4 h}{\sqrt{a_1^8 - 4a_2^8}}$$

Also for x given in (5.1) $\frac{d^2P}{dx^2} > 0$

Hence, equation (5.1) gives the bifurcation point x

From above figure, we have

$$\text{(5.2)} \quad \cos(\theta/2) = \frac{L-x}{\sqrt{h^2 + (L-x)^2}}$$

$$\text{(5.3)} \quad \frac{\theta}{2} = \cos^{1}\left(\frac{L-x}{\sqrt{h^2 + (L-x)^2}}\right)$$

Since, $0 \le x \le L$, then

$$\text{(5.4)} \quad \cos^{-1}\left(\frac{L-x}{\sqrt{h^2 + L^2}}\right) \le \frac{\theta}{2} \le \frac{\pi}{2}$$

Combining of equation (5.1) and (5.2), we obtain for minimum value of P

$$\text{(5.5)} \quad \cos\left(\frac{\theta}{2}\right) = 2\left(\frac{a_2}{a_1}\right)^4$$

Thus, from (5.4) and (5.5), we get a condition that P will be minimum if

$$2\left(\frac{a_2}{a_1}\right)^4 \ge \frac{L}{\sqrt{h^2 + L^2}}$$

Corresponding to $a_2 = 0$, $x = L$ and □ = □ as a_2 increases the bifurcation point x moves to the left and the bifurcation angle □ until x = 0, when

$$a_2 = a_1 \left[\frac{L^2}{4(h^2 + L^2)}\right]^{1/8}$$

and $$\frac{\theta}{2} = \cos^{-1}\left(\frac{L}{\sqrt{h^2 + L^2}}\right)$$

6. Application

Our study and analysis made in this paper are relevant in biomechanics and are very useful in medical science specially in heart attack due to improper blood flow in artery and some other biomedical investigations.

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